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A nearly optimal auction for an uninformed seller[☆]

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HIGHLIGHTS

- The seller does not know the distribution of values of potential buyers.
- Our selling mechanism combines an elicitation mechanism with a standard auction.
- The uninformed seller achieves nearly the same expected revenue as Myerson (1981).

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ABSTRACT

This paper describes a nearly optimal auction mechanism that does not require previous knowledge of the distribution of values of potential buyers. The mechanism we propose builds on the new literature on the elicitation of information from experts. We extend the latter to the case where the secret information shared by the experts – potential buyers in our model – can be used against them if it becomes public knowledge.

1. Introduction

The optimal mechanism to sell a single object requires prior knowledge of the distributions of values of potential buyers.[1] In the symmetric independent private value model, for instance, the optimal direct mechanism can be obtained by using a second price sealed bid auction with a specific reservation price. The reservation price depends on the distribution of bidder values. The construction of an optimal auction for the asymmetric case is similarly tethered to the seller's knowledge of the value distribution of each individual bidder. What if the seller does not know these distributions? We provide an almost optimal mechanism for an uninformed seller in a context where the group of potential buyers are aware of the value distributions. In terms of Krishna (2010), our mechanism has the advantage of being "detail-free".

The nearly optimal auction we propose consists of two mechanisms: an elicitation mechanism and an auction. The aim of the elicitation mechanism is to recover the distributions of values of the potential buyers whereas the aim of the auction is to maximize expected revenue. These two mechanisms are intimately related. The details of the auction depend on the distributions obtained from the bidders in the elicitation mechanism, and the lottery payoffs of the elicitation mechanism depend on the bids placed by the potential buyers in the auction. Despite the informational disadvantage placed on the seller, the auction we propose almost always obtains the maximal expected revenue for the seller at a near zero cost. Moreover, the induced game among potential buyers is individually rational and strict incentive compatible.

The elicitation part of our mechanism builds on the recent literature about information elicitation from experts. Karni (2009) introduces an incentive compatible mechanism for eliciting the subjective probabilities of an agent about a finite number of

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[1] See Laffont and Maskin (1980), Myerson (1981), or Riley and Samuelson (1981).

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events.[2] Qu (2012) extends Karni's mechanism to the elicitation of an agent's beliefs about the general distribution of a random variable. These papers assume that the expert has no stake in the random behavior of interest.[3] In our model, the information disclosed by the experts – the bidders in the auction – will be used by the seller against them. We show that Karni and Qu's contributions can be extended to this delicate situation whenever there are at least two experts or bidders in our context. In this sense, our approach formalizes the well-known phrase.

A secret between more than two is not a secret.

There are a few other papers related to our idea. The recent literature on the econometrics of optimal auctions solves the problem of the uninformed seller by using a sequential auction mechanism (see, e.g., Paarsch, 1997). That is, this literature assumes that the seller runs (or has run) an initial auction, obtains some data, and then uses this information to recover the value distributions of the bidders. The seller then computes and conducts an optimal *new* auction for subsequent units of the good. This procedure is costly in terms of foregone revenue and may not be practical if the seller has a unique item and cannot therefore take advantage of the obtained information. Segal (2003) addresses the same issue by providing a mechanism that sets a price for each buyer on the basis of the demand distribution inferred statistically from other buyers' bids.[4] The resulting profit converges to the optimal monopoly profit with known demand as the number of buyers goes to infinity. We assume that consumers know the distribution of valuations. Under this condition, the advantage of our approach over the one of Segal is that, in our set-up, profits are almost optimal even when the number of bidders is very small (i.e., two bidders in the symmetric model and three bidders in the asymmetric one). Brooks (2013) has recently considered a similar problem but proposed a very different solution. Our mechanism is simpler and the seller controls (via the lottery prizes) the maximum cost he could incur to recover the valuations of potential consumers. On the other hand, Brooks' mechanism allows for certain type of correlation. The project finally relates to the newer literature on the robust mechanism design (see, e.g., Bergemann and Morris, 2005 and Börges, 2013). The latter builds on the observation that the mechanism design literature assumes too much common knowledge of the environment among the players and planner and aims at relaxing this restriction. We keep common knowledge of the environment among the players but relax the information requirement often imposed on the seller.

2. Almost optimal mechanism for the symmetric model

We model a situation in which the seller has a single good for sale and there are $n \geq 2$ potential buyers with quasi-linear preferences for the object. Bidder i assigns a value x_i to the item. Each bidder's value is unknown to the seller and to the other bidders. Their values are independent and identically distributed according to a cumulative distribution function $F : [\underline{x}, \bar{x}] \to \mathbb{R}_+$ with $-\infty < \underline{x} < \bar{x} < \infty$. The probability density function of F, f, is continuous and strictly positive everywhere on $[\underline{x}, \bar{x}]$. The problem is *regular* in the sense that the virtual valuation function

$$\Psi_F(x) = x - \frac{[1 - F(x)]}{f(x)}$$

is strictly increasing in x. The potential buyers are aware of the distribution of values and this awareness is common knowledge. Our modeling assumptions differ from the standard ones in that the seller does not know F.

The goal of the seller is to maximize expected profits. If the seller knew F, then a second price auction with reserve price r^* implicitly defined by $\Psi_F(r^*) = 0$ would be the optimal direct mechanism. However, in our model, the seller does not know F and, hence, cannot directly set an optimal reservation price. We now provide a mechanism in which the seller elicits F from the potential buyers, at an almost zero cost, and uses this information to implement a second price auction with a reservation price that is almost always equal to r^*.

The nearly optimal auction we propose consists of two interrelated mechanisms: an elicitation mechanism and a standard auction. The elicitation mechanism is, essentially, a stochastic Vickrey–Clarke–Groves (VCG) mechanism conducted between each bidder and a dummy bidder for a lottery payoff. In the auction mechanism, the item is allocated according to a standard second price auction with a stochastic reserve price. The two mechanisms are intimately related: first, the reserve price in the auction depends on the distributions obtained from the bidders in the elicitation mechanism. Second, the lottery payoffs of the elicitation mechanism depend on the bids placed by the bidders in the auction. The mechanism we offer provides strong incentives for each bidder to report truthfully both own valuation and the distribution of values if it is believed that at least one other will do so. Specifically, truthfully report of distributions and values is a strict Bayesian Nash equilibrium of the induced game.

The rules of the game are as follows. Each agent i submits a message to the seller containing two pieces of information: a nonnegative bid, b_i, and a cumulative distribution function, G_i. The seller takes these messages and, for each i, computes the largest root of $\Psi_{G_i}(x) = 0$ that we indicate by r_i. If $\Psi_{G_i}(x)$ has no root, then the seller sets $r_i = 0$. Thus, we can think of r_i as the reserve price suggested by bidder i. Then the seller draws a random vector (p, t, k). It is common knowledge that p and t are *i.i.d.* draws from the uniform distribution on $[0, 1]$ and k is a random realization from a distribution H with full support on $(-\infty, \infty)$. The realization p is used to set-up the reserve price in the auction and the numbers (t, k) are used in the elicitation mechanism. We next formalize the two related mechanisms.

Auction mechanism: the good is allocated according to a second price auction with a stochastic reserve price given by

$$r = \begin{cases} \max\{r_1, r_2, \ldots, r_n\} & \text{if } p \leq \bar{p} \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{p}$ is known by all the bidders. Thus, $\bar{p}$ is the probability that the reserve price be equal to $\max\{r_1, r_2, \ldots, r_n\}$ and the probability that r be equal to zero is just $1 - \bar{p}$. Once r is defined, the item is allocated according to the usual rule. The fact that the reserve price is zero with strictly positive probability guarantees that each bidder has strict incentives to report his value even when it is very small.

Elicitation mechanism: each bidder in the auction enters into a lottery for a chance to win a prize $w > 0$. For bidder i, the lottery depends on both G_i and (t, k). It is determined as follows: let E_i be the event that bidder $i + 1$'s bid, b_{i+1}, falls in the region $(-\infty, k]$, with $n + 1 \equiv 1$. We define $L_i(t, w)$ as a lottery where bidder i wins the prize w with probability t and it wins 0 with probability $1 - t$. If bidder i submits that the distribution G_i and (t, k) are realized, then he receives the following lottery payoff

$$m_i(G_i, t, k) = \begin{cases} w\,1(E_i) & \text{if } G_i(k) \geq t \\ L_i(t, w) & \text{otherwise} \end{cases}$$

where $1(.)$ is the indicator function that takes a value of 1 if the event E_i occurs and 0 otherwise. This lottery mechanism is similar to the mechanisms proposed by Qu (2012) in the context of eliciting probability distributions from experts.

2 This mechanism is related to the elicitation procedure presented by Becker et al. (1964). Recently, Demuynck (2013) illustrated how Karni's mechanism may be used for eliciting the mean or quantiles of a random variable.

3 O'Hagan et al. (2006) survey this literature which spans several fields.

4 See also Hartline (2012) who discusses approximation in the mechanism design and surveys some of this new literature.

The next proposition states that, in the proposed mechanism, submitting own values and F is a strict Bayesian Nash equilibrium.

Proposition 1. *For any $w > 0$, the strategy profile in which each bidder i submits $b_i = x_i$ and $G_i = F$ is a strict Bayesian Nash equilibrium for the nearly optimal auction. In addition, this mechanism is individually rational.*

Proof. Let us assume that each bidder $j \neq i$ submits $b_j = x_j$ and $G_j = F$. Let r satisfy $\Psi_F(r) = 0$. We next show that $b_i = x_i$ and $G_i = F$ is a strict best-response for bidder i. Independent of G_i, $b_i = x_i$ is clearly always optimal for agent i. Moreover, since the reserve price is zero with strictly positive probability, bidder i has strict incentives to report $b_i = x_i$ even if $x_i < r$. Therefore, for each reserve price r, the expected profits of bidder i (if he reports $b_i = x_i$) are given by

$$u_i(x_i, G_i) = \begin{cases} y_i(G_i) & \text{if } x_i \leq r \\ x_i F(x_i)^{n-1} - \left[rF(r)^{n-1} + \int_{r(G_i)}^{x_i} z(n-1)f(z)F(z)^{N-2}dz \right] + y_i(G_i) & \text{if } x_i > r \end{cases}$$

where $y_i(G_i)$ is the expected income determined by the lottery.

Recall that we assumed all bidders different from i report F.[5] By choosing G_i, bidder i can only distort r up. This type of behavior can never increase bidder i's expected payoff. We next show that it is strictly optimal for i to submit F as well.

Assume that bidder i submits G_i. Then, for each realization k, his expected payoff from the lottery is given by

$$\begin{aligned} \mathrm{E}_t\left[m_i(G_i, t, k)\right] &= G_i(k)wF(k) + (1 - G_i(k)) \\ &\quad \times \int_{G_i(k)}^{1} wt/(1 - G_i(k))\, dt \\ &= G_i(k)wF(k) + w/2 - w\left(G_i(k)\right)^2/2. \end{aligned}$$

This function is strictly concave in $G_i(k)$. From the first order condition, the strict global maximum is $G_i(k) = F(k)$. Since this is true for all k and H has full support on all $(-\infty, \infty)$, submitting F is a strict best response for bidder i.

The fact that the mechanism is individually rational holds as if agent i bids his value, then his smallest possible payoff is 0. ■

As we mentioned earlier, if the seller knew F, then a second price auction with reservation price r^* implicitly defined by $\Psi_F(r^*) = 0$ would be the optimal direct mechanism. The mechanism we have proposed for the uninformed seller is nearly optimal for two reasons: first, by choosing $\bar{p}$ close to one, the seller will almost always implement the optimal auction. Second, the maximum possible payment he can incur ($nw > 0$) can be made arbitrarily small.[6] As a practical concern, however, there is some question about how bidders would submit a distribution to an auctioneer. While we do not address this issue directly, we can report that there is a literature on this problem and that procedures for eliciting distributions have been developed and used in practice (see, e.g., O'Hagan et al., 2006).

The next section shows that our proposed mechanism can be easily extended to the case of asymmetric distributions if there are at least three potential buyers. Before doing so, we explain that for the symmetric model there is a simpler mechanism that achieves the same result. This mechanism does not require bidders to submit distributions to the seller.

An alternative mechanism: in the symmetric model, a nearly optimal mechanism can be constructed using a simpler message space. Specifically, we next show that we can substitute the requirement to submit full distributions to the seller for one in which agents simply report bids and reservation prices.[7] Let us consider a mechanism where agent i submits a message to the seller containing a non-negative bid, b_i, and a non-negative suggested reservation price r_i—this is just a simple descriptive statistic of the underline distribution. The seller allocates the item according a second price auction with a stochastic reserve r as previously defined. In addition, each bidder enters a lottery for a prize wr_i. This prize is awarded to agent i in the event that bidder $i+1$'s bid, b_{i+1}, falls in the region $[r_i, \infty)$, and it is otherwise equal to zero. As before, we define $n + 1 \equiv 1$.

We next explain that the message profile where all bidders submit their true values and the optimal reserve price r^* is a strict Bayesian Nash equilibrium of this auction as well. Suppose each bidder $j \neq i$ chooses according to the suggested Bayesian Nash equilibrium. Then, selecting $b_i = x_i$ is strictly optimal for agent i. Since bidder $i+1$ is reporting truthfully, from the perspective of bidder i, his bid is distributed according to F. Therefore i's expected lottery payoff from reporting r_i is equal to $wr_i(1 - F(r_i))$. This expected payoff is clearly maximized at $r_i = r^*$. Finally, for all $j \neq i$, we have $r_j = r^*$. It follows that agent i can only affect r by choosing $r_i > r^*$. This would lower both agent i's expected auction payoff and his expected lottery payoff. Hence, $(b_i, r_i) = (x_i, r^*)$ is a strict best response and the result is established.

Unfortunately, the simpler mechanism we just discussed does not directly extend to the situation where bidder's types are asymmetrically distributed. The next section shows that the opposite is true regarding our initial mechanism. The only relevant difference is that for the following case we need three bidders, as compared to only two, for the mechanism to work.

3. Almost optimal mechanism for the asymmetric model

The main idea in our proposed mechanism can be used to derive a nearly optimal auction for an uniformed seller facing $n \geq 3$ heterogeneous bidders, where the value for the item of each bidder i derives from the distribution F_i on $[\underline{x}, \bar{x}]$. Here again, we assume that $\Psi_{F_i}(.)$ is strictly increasing for all $i \leq n$.

The rules of the game are as follows: each bidder i simultaneously submits a type b_i and a cumulative distribution function G_{ij} for each $j \neq i$ to the seller. The seller takes these messages and, for each i, computes the price

$$p_i(\mathbf{b}_{-i}) = \inf\{z_i | \tilde{\Psi}_i(z_i) \geq \tilde{\Psi}_j(b_j) \text{ for all } j \leq n, j \neq i \text{ and } \tilde{\Psi}_i(z_i) \geq 0\},$$

where $\mathbf{b}_{-i} = (b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n)$ and

$$\tilde{\Psi}_i(x) = \max\{\Psi_{G_{1i}}(x), \ldots, \Psi_{G_{(i-1)i}}(x), \Psi_{G_{(i+1)i}}(x), \ldots, \Psi_{G_{ni}}(x)\} \text{ for all } x \in [\underline{x}, \bar{x}].$$

In addition, the seller draws a random vector (p, t, k) as in the previous section.

Auction mechanism: the good is allocated via a simple stochastic auction. Specifically, if $p \leq \bar{p} < 1$, then the good is assigned to bidder i only if $\tilde{\Psi}_i(b_i) > \max_{j \leq n, j \neq i} \tilde{\Psi}_j(b_j)$ and $\tilde{\Psi}_i(b_i) \geq 0$ in which case he pays $p_i(\mathbf{b}_{-i})$. If bidder i does not get the item, then he pays 0. On

[5] Note this argument holds if even if only one other bidder has submitted truthfully.

[6] Note the seller could reduce the variance of realized lottery payoffs by running independent lotteries across agents—i.e., by computing independent random draws (t_i, k_i) for each bidder i.

[7] We are grateful to an anonymous referee for suggesting this mechanism to us.

the other hand, if $p > \bar{p}$, then the good is allocated according to a second price auction using the submitted $\mathbf{b} = (b_1, b_2, \ldots, b_n)$ as the bids. As before, we assume that $\bar{p}$ is known by all the bidders.

Elicitation mechanism: upon submitting profiles, all the bidders receive their payoffs from participating in the auction and enter into $n - 1$ lotteries each of which provides the bidders a chance to win a payment $w > 0$. For each $j \neq i$, the lottery for bidder i depends on G_{ij} and (t, k). Let E_{ij} be the event that bidder j's reported type b_j falls in the region $(-\infty, k]$. We define $L_{ij}(t, w)$ as a lottery where bidder i wins the prize w with probability t and he wins 0 with probability $1 - t$. If bidder i submits the distribution G_{ij} and (t, k) are realized, then he receives the following lottery payoff

$$m_{ij}\left(G_{ij}, t, k\right) = \begin{cases} w1(E_{ij}) & \text{if } G_{ij}(k) \geq t \\ L_{ij}(t, w) & \text{otherwise} \end{cases}$$

where $1(.)$ is the indicator function that takes a value of 1 if the event E_{ij} occurs and 0 otherwise.

Truth telling is also a strict Bayesian Nash equilibrium of this auction. To see this, suppose each bidder $j \neq i$ reports truthfully (i.e., each bidder reports his type and the true value distributions of the other bidders to the seller). Now bidder i can only increase $\tilde{\Psi}_j(x_j)$ with his report of G_{ij}. Doing so is not optimal as it increases his payment when he wins the auction and decreases his chances for winning. In addition, since each lottery is structured as the one in Proposition 1, he has strict incentives to report the true value distributions—i.e., $G_{ij} = F_j$ for all $j \neq i$. Hence, we get that $\tilde{\Psi}_i = \Psi_i$ for all i. The realized auction is therefore either Myerson's 1981 optimal auction or, with a very small but positive probability, a second price auction without a reserve price. It follows that i's strict optimal bid is $b_i = x_i$. Finally, since w can be any positive number, the seller total cost of extracting information from the bidders can be made arbitrarily small by choosing a sufficiently small prize w.

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